STATEMENT REPORT

under the procedure for acquisition of **the educational and scientific degree "Doctor"**
by candidate **Martin Jordanov Minchev,**
of the PhD Thesis entitled: "Functionals of Lévy processes and their applications",
In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**
Professional field: **4.5. Mathematics**
Doctoral program "Probability Theory and Mathematical Statistics", Department „ Probability, Operations Research and Statistics",
Faculty of Mathematics and Informatics (FMI), **Sofia University "St. Kl. Ohridski" (SU),**

The statement report has been prepared by: assoc. prof. Vessela Kirilova Stoimenova, PhD, as a member of the scientific jury for the defense of this PhD thesis according to Order № РД 38-200 / 26.04.2024 of the Rector of the Sofia University.

1. **General characteristics of the dissertation thesis and the presented materials**
   The PhD Thesis has a volume of 175 pages and consists of an introduction, four chapters, a conclusion, two appendices and a bibliography numbering 173 titles. It is written in English.

2. **Short CV and personal impressions of the candidate**
   Martin Minchev completed his secondary education at Sofia Mathematical High School "Paisiy Hilendarski". He obtained his bachelor's degree in applied mathematics at Sofia University "St. Kliment Ohridski" in 2018, and the master's degree in probability and stochastic models - at the Sorbonne University, Faculty of Sciences and Engineering, Paris, France, in 2019. He is enrolled in full-time doctoral studies at the Department of Probability, Operations Research and Statistics of the FMI, SU, in October 2020.
   My personal impressions of the PhD student are very good. I had the opportunity to follow his development during the years of his doctoral studies at the department, I witnessed the hours of work put in, his diligence, perseverance and responsibility towards his duties as a doctoral student and the fulfillment of his assigned tasks. Outside of scientific work, Martin Minchev leads exercises for the department as a part-time assistant professor in the disciplines Probability and Statistics, Statistics and Empirical Methods, Probability Theory and Mathematical Statistics, Random Processes 1 and 2. He treats these duties with the same dedication and responsibility, contributing to the improvement and enrichment of the courses.

3. **Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure**
   The main scientific contributions of the author are placed in chapters IV and V of the work. Results on the probability density and its smoothness and tail behavior of the distribution of the exponential functional of subordinators, as well as an improvement of the Stirling asymptotics of the Bernstein-gamma functions, are included in Chapter IV, respectively in Section 17 (Theorem 17.1 and Corollaries 17.2, 17.4 , 17.6) and in Section 19 (Theorem 19.3, Lemmas 19.1 and 19.8, and Corollary 19.5). Their proofs are given in a separate Section 20. These results are published in the 2023 paper by Minchev and Savov, presented

in the candidates documents. Chapter V is dedicated to the study of the weak convergence of scaled exponential functionals of a Lévy process with a finite negative mean and a regularly varying tail (Theorem 23.1). As a consequence, asymptotics for the expectation of their functional transformations are derived. Section 24 of the same chapter analyzes the two-dimensional Bernstein-gamma functions through the link to the potential measure of the Lévy process and its convolutions. Detailed proofs are presented in Sections 25 and 26, which include necessary additional statements, interesting and valuable in themselves.

The introduction (referred to as Chapter I) introduces Lévy process theory. Chapter II gives basic statements related to the exponential functionals of the Lévy process (with finite, infinite, or random exponentially distributed horizon), and the relation to Ornstein-Uhlenbeck processes and self-similar Markov processes. Chapter III presents results related to Bernstein-gamma functions. These initial three chapters of the PhD Thesis represent a comprehensive, systematized, and useful overview of the existing results in the field.

4. Approbation of the results

The results of Chapter IV have been published in a joint article by the PhD student and his supervisor Prof. Savov in the journal Bernoulli, IF 1.5 for 2022, Quartile Q2 (Statistics and Probability). There are 7 citations noticed. A declaration of co-authorship is provided indicating that the contributions of the two co-authors are equal.

PhD Thesis results are presented at the Stochastic Processes and their Applications conferences, Lisbon, Portugal, July 24-28, 2023; Mathematics Days in Sofia, July 10-14, 2023, Sofia, Bulgaria; Lévy Processes and Random Walks (in celebration of Ron Doney's 80th birthday), 26-28 July 2022, Manchester, UK; FMI Spring Scientific Session, March 26, 2022, Sofia, Bulgaria, and at the Seminar on Advances in Statistics, March 9-12, 2023, Veliko Tarnovo, Bulgaria.

The plagiarism check (Originality Check Protocol attached to the documents) shows that the PhD student's results are original and there is no plagiarism.

The scientific works meet the minimum national requirements (under Art. 2b, para. 2 and 3 of ADASRB*) and respectively to the additional requirements of Sofia University "St. Kliment Ohridski" for acquiring the educational and scientific degree "Doctor" in the scientific field and professional field of the procedure.

5. Qualities of the abstract

The abstract is written in Bulgarian (56 pages) and in English (57 pages) and is in the required volume and format. The content of the dissertation, the contributions of the doctoral student and the approval of the results are correctly described in it.

6. Critical notes and recommendations

The field of dissertation work has its specifics such as terminology and notations. Although the doctoral student has tried at the beginning of each chapter to recall the necessary notation, it would be useful to present a list of the notations used at the beginning of the work, which would further facilitate the understanding of the thesis. The dissertation uses an atypical numbering of chapters and sections that have no relation to each other. Statements follow section numbering, but there is no separate numbering for statement and comment (remark). For example, after Corollary 17.4, Remark 17.5 is given, followed by Corollary 17.6, and the

reader is left with the impression that he has missed the non-existent Corollary 17.5. Likewise, an atypical numbering of the bibliography has been adopted, which is puzzling on initial reading. It would be good if the structure of the work and the numbering used are explained at the beginning of the dissertation to avoid ambiguities. There are also some technical errors in the text, especially in the author's reference in Bulgarian. At the beginning of the proof sections, it would be appropriate to outline the main steps and explain their necessity before proceeding to the long detailed descriptions. These remarks and recommendations do not fundamentally change my good impressions of the dissertation and the candidate's scientific knowledge of the topic of the dissertation work.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publication to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I recommend** the scientific jury to award Martin Jordanov Minchev the educational and scientific degree "Doctor" in the Scientific field 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics (Doctoral program "Probability Theory and Mathematical Statistics", Department „ Probability, Operations Research and Statistics", Faculty of Mathematics and Informatics, Sofia University "St. Kl. Ohridski").

Date: 05.07.2024

Signature: ....
/assoc. prof. Vessela Stoimenova, PhD /